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with each other with all desirable accuracy; and yet the ratios indicated by them are very much greater than those of 1853 and 1854. He adds, that it would be useless to speculate at present upon the cause of this discordance; but he looks forward with interest to the observations which he hopes to make at the present opposition in the expectation of obtaining some solution of the difficulty. He concludes with the following remarks:—

"For the present, I can only express my opinion that the results which are given exhibit with tolerable accuracy the dimensions of the *Saturnian* system, and that no change whatever has taken place in the system since the time of Huygens. If this opinion be correct, it will follow that M. Struve's measures of the breadth of the dark space between the bright rings and the ball have been influenced by some circumstance which has not affected the distance between the ball and the outer edge of the outer ring, for in this latter particular my measures are almost coincident with his. Under these circumstances there is little doubt that M. Struve will be anxious to seize the favourable opportunity that presents itself at the present opposition of the planet for repeating his measures in a way that will prevent the possibility of doubt; and I may at the same time direct the attention of such English astronomers as are furnished with good telescopes equatoreally mounted to the interesting circumstances which I have attempted in the present memoir to elucidate."

On the Attraction of the Himalaya Mountains and of the elevated Regions beyond them, upon the Plumb-line in India. By the Venerable John Henry Pratt, M.A., Archdeacon of Calcutta.*

The author of this paper commences by remarking that the results of the measurement of the great arc of the meridian of India afford undoubted indications of the disturbing influence exercised upon the direction of the plumb-line by the attraction of the Himalaya Mountains and the elevated regions lying beyond them. Thus, the amplitude of the northern division of the arc included between Kaliana and Kalianpur, when determined by astronomical observations of latitude at the two extreme stations, was found to be 5° 23′ 37″·058; whereas, when computed geodetically, assuming the usually admitted values of the major axis and ellipticity of the earth, the value of the amplitude appeared to be 5° 23′ 42″·294. The circumstance of the astronomical arc being in *defect* as compared with the geodetical arc, is favourable to the supposition of the discordance being due to the attraction of the mountainous country situate to the north of the arc; for the ten-

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* Phil. Trans., 1855, p. 53, *et seq.*

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dency of the disturbing attraction in such a case would manifestly be to produce a greater deviation of the plumb-line at the northern than at the southern extremity of the arc, and thereby to diminish the astronomical determination of the amplitude of the arc somewhat below its true value. The question, however, arises, is the *quantity* of this discordance, namely, $5''{\cdot}236$, the same as that which would be occasioned by the disturbing influence exercised upon the direction of the plumb-line at the two stations? The author has mainly devoted his paper to the discussion of this important point.

The geodetical operations cannot be considered as wholly exempt from the influence of mountain attraction, seeing that in determining the elevations and depressions of the stations of the various triangles, the direction of the plumb-line must be taken into account. The author, however, has shown (what indeed could scarcely have required demonstration) that in consequence of the smallness of the elevations and depressions the effect thus produced is utterly insensible. He next investigates the effect which a small error in the amplitude of an arc would produce on the value of the ellipticity; and he finds, that by assuming the ellipticity to be $\frac{1}{300}$, an error in defect of $5''{\cdot}236$ in the amplitude would, under the most favourable circumstances, diminish the ellipticity by $\frac{1}{25}$th of its whole value. It is not, therefore, sufficient to attribute the discordance to the influence of mountain attraction; but, on the contrary, it becomes an object of the highest importance to ascertain whether the effects due to this cause are capable of accounting exactly for the observed anomaly.

In order to calculate the attraction at any station, A, of the superincumbent matter contained on the earth's surface, the author supposes a series of vertical planes to be drawn through A, making any angles with each other, and dividing the surface through A parallel to the sea-level, into a number of *lunes*, all meeting again in a point at the antipodes of A. He then draws a series of concentric circles upon this surface, having A for their common centre, the distance between two consecutive circles being regulated by a fixed law. In this way he divides the whole surface into a number of four-sided *compartments*, two of the sides in every compartment converging to A, and the other two being parts of circles concentric with respect to A.

Supposing β to be the angle contained between any two vertical planes passing through A, and which consequently measures the breadth of the lune formed by them; α and $\alpha + \varphi$, the angular distances from A of any compartment in this lune; ϱ, the mean density of the earth; and h, the average height of the earth's surface above the level of the compartment; the author finds the attraction upon A of the whole mass standing on the compartment to be

$$= \varrho \sin \tfrac{1}{2}\beta \,.\, \frac{\varphi \cos^2\left(\frac{1}{2}\alpha + \frac{1}{4}\varphi\right)}{\sin\left(\frac{1}{2}\alpha + \frac{1}{4}\varphi\right)} \,.\, h$$

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This expression is simplified by supposing the relation between α and ϕ to be such that

$$\frac{\phi \cos^2 (\frac{1}{2}\alpha + \frac{1}{4}\phi)}{\sin (\frac{1}{2}\alpha + \frac{1}{4}\phi)} = \text{a numerical constant} = c$$

The value of c being determined by making $\phi = \frac{1}{10}\alpha$, ϕ and α being both supposed indefinitely small, the relation between ϕ and α is reduced to the following form:

$$\frac{\phi \cos^2 (\frac{1}{2}\alpha + \frac{1}{4}\phi)}{\sin (\frac{1}{2}\alpha + \frac{1}{4}\phi)} = \frac{4}{21}$$

whence attraction of mass standing on any compartment

$$= \frac{4}{21}\,\varrho \sin \tfrac{1}{2}\beta \,.\, h$$

This expression depends upon h, the average height of the surface of the mass, above the surface through A, and not at all upon the distance of the compartment from A.

The equation between ϕ and α obviously fixes the relation between the distances from A of any two consecutive circles, the arcs of which form the sides of the compartments; or, in other words, the relation between the length of any compartment and its distance from A. The author calls this relation the *Law of Dissection.*

It will be seen that the expression for calculating the attraction of the matter above the surface passing through A is reduced to a formula of extreme simplicity. It only remains to calculate the distances from A by means of the Law of Dissection, and lay down the circles and lines diverging from A upon a good map on which the elevations and depressions are marked, and the attractions of the several masses standing on the compartments thus marked out will be given by the formula for the attraction as soon as their average elevations have been determined.

Assuming the mean density of the earth to be 5·66 times the density of distilled water as determined by Baily, and supposing the density of the superficial crust of the earth to be = 2·75, which is the density assigned to the mountain Schehallien, the author finds the deflection of the plumb-line caused by the mass standing on any one compartment to be

$$= 1''\cdot 1392 \sin \tfrac{1}{2}\beta \,.\, h$$

The equation between ϕ and α involving the Law of Dissection, not admitting of a direct solution, the author employs an ingenious process by which he is enabled to effect this object with all desirable accuracy, and hence he derives the lengths of the compartments and their respective distances from A. These distances being laid down and the concentric circles drawn upon a map or globe, it only remains to ascertain the average heights of the masses standing on the compartments thus drawn. The author

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suggests some practical methods for effecting this object, and concludes the exposition of his method of computation with the following remark:—

"It will be evident that mountain ranges will assume a less importance in this calculation as they are more distant from A, since they will stand on a larger compartment; and therefore, when in imagination levelled down to cover the whole compartment, and give the average height, they will stand at a much less altitude. It is for this reason that a knowledge of extensive table-lands of considerable elevation, and of the elevated channels of rivers is of far more importance in this part of the calculation, especially in the remoter parts, than of mountain-peaks and mountain-ridges."

The author next investigates an approximate value of the intensity of mountain attraction at the two extremities and the middle station of the arc. The extreme stations are Kaliana, in latitude 29° 30′ 48″ and Damargida, in latitude 18° 3′ 15″; the middle station is Kalianpur, in latitude 24° 7′ 11″. The northern, middle, and southern stations are denominated by him respectively A, B, and C.

He first traces out upon a map the tract of country which he supposes to exercise a sensible influence upon the direction of the plumb-line in India. This he calls the *Enclosed Space.* Before describing the geographical features of this region, he shows, by reference to Humboldt's *Aspects of Nature* and other authorities, that the attraction of the countries situate externally with respect to it cannot sensibly affect the direction of the plumb-line at any station of the great arc of India. He then divides the Enclosed Space into two regions, which he denominates respectively the *Known Region* and the *Doubtful Region.* The general features of the Known Region are gathered from the Survey and other maps of India. With respect to the Doubtful Region the author is compelled to rely upon the statements of Humboldt and other travellers. Having divided the Enclosed Space into a number of four-sided compartments by drawing a series of lines and concentric circles corresponding to each of the three stations, in accordance with the method of computation which he had already explained, he endeavours to determine the average height on each compartment by means of the knowledge thus derived respecting its geographical features. The computation of the attraction at each station was then effected with great facility. The following are the final results at which he arrived:—

				″
Deflection of plumb-line in	meridian	at A	..	27·853
,,	,,	at B	..	11·968
,,	,,	at C	..	6·909
,,	in prime vertical	at A	..	16·942
,,	,,	at B	..	4·763
,,	,,	at C	..	2·723

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Hence we obtain

	″
Difference of Meridian Deflections at A and B ..	15·885
,, ,, at A and C ..	20·944
,, ,, at B and C ..	5·059

The first of these quantities is considerably greater than 5″·236, the quantity brought to light by the *India Survey;* and the values of the deflections at B and C bear a far higher ratio to those at A than has been generally supposed. The following are the results for the total deflection at each of the stations:—

		″		° ′	
Total Deflection at A ..		32·601,	and in Azimuth,	31 18	east.
,, ,,	B ..	12·880	,,	21 42	,,
,, ,,	C ..	7·426	,,	21 31	,,

The author next makes various suppositions for reducing the computed deflection so as to make it agree more nearly with the results of observation. His original assumption was that the density of the elevated region is 2·75 times the density of distilled water, which was the estimated density of the mountain of Schehallien. His opinion is that this assumption is not too great, since a large share of the deflection is produced by the attraction of the elevated plateau which lies in Thibet and south of that country; and as this is on an average 10,000 feet or more high, the lower part of the materials must be denser rather than lighter than those of a mountain of inconsiderable altitude. "If, however," continues the author, "we do reduce the density, say to 2·25, which is yielding much, still the deflections and their differences are reduced by only one-fifth part, and therefore this will not solve the difficulty."

The author next has recourse to an hypothesis which is tantamount to supposing the whole mass on the Doubtful Region to be non-existent; but even in this case the difference of deflection at A and B is not reduced below 9″·753, which is greater than 5″·236 in the ratio of 13 to 7. Nor will the attraction be sufficiently diminished if the density of the remaining mass on the Known Region be reduced.

A third means of reduction founded upon the supposition of the non-existence of the mass on the Doubtful Region, and a diminution of the altitude of the mass on the Known Region is found to be equally ineffectual in reconciling the computed with the observed deflection.

By calculation the author finds

$$\frac{\text{Mass on Doubtful Region}}{\text{Mass on Known Region}} = 6{\cdot}264$$

or the mass on the Doubtful Region is greater than that on the Known Region in a ratio higher than 25 to 4. Also,

$$\frac{\text{Mass on whole Enclosed Space}}{\text{Mass of the Earth}} = 0{\cdot}000028257$$

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The author next computes for each station the point at which the whole of the elevated mass would require to be concentrated in order that it might produce the same amount of deflection. In this manner he finds the

		Miles.
Distance from A of point of concentration	=	1688
.. B	=	2692
.. C	=	3544

The differences of these two last from the first is far greater than the distance of B and C from A, viz. 5° 23′ 37″ and 11° 27′ 33″. From this it is easily inferred, what indeed did not need this proof, that the mass in no sense whatever, even an approximate one, attracts as if concentrated in a fixed point.

Assuming that the heights of the elevated region have been rightly assigned in his paper, the author finds by a comparison of the northern and southern divisions of the Indian arc, that its ellipticity amounts to $\frac{1}{426\cdot 2}$, a result which shows that the arc is more curved than it would be if it had the mean ellipticity. The author illustrates the increased curvature by comparing the height of the middle point of the arc above the chord with the height of the same point furnished by the mean ellipticity $(\frac{1}{300})$ and by the ellipticity which has been deduced from the Indian arc without taking into account the effect of mountain attraction, viz. $\frac{1}{196\cdot 3}$. In this way he finds

		Miles.
For the mean ellipticity, height	=	19·8992
Ellipticity $\frac{1}{426\cdot 2}$		19·9290
.. $\frac{1}{196\cdot 3}$		19·8460

“The ellipticity, therefore,” says the author, “which results from taking account of mountain attraction raises the middle point of the arc by 0·0298 of a mile, or 157 feet; whereas the ellipticity, when mountain attraction is neglected, depresses the arc through 0·0532 of a mile, or 281 feet. These quantities are nearly in the ratio of 5 to 9. Hence the consideration given to mountain attraction in this paper brings the curvature of the Indian arc nearer the mean curvature than the neglect of mountain attraction does in the ratio of 5 to 9. This is, as far as it goes, in favour of these calculations.

Deep River, Cape of Good Hope,
July 12, 1854.”